BRIEF COMMUNICATIONS

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Nonlinear dynamics and transient growth of driven contact lines

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We consider the theory of driven contact lines in a complete wetting scenario and examine the effect of small scale localized surface disturbances on the global shape of the film profile. We compute how the nonlinear amplification scales with the precursor thickness of the film and the characteristic width of the surface pattern. Nonlinear disturbances of the film profile are connected to ''transient growth'' in the linear stability analysis [Phys. Fluids **9**, 530 (1997)]. © *1999 American Institute of Physics.* [S1070-6631(99)02711-7]

The stability of driven contact lines has an extensive literature, both experimentally[1–3] and theoretically.[4–8] The problem of gravity driven contact lines (flow on an inclined plane) poses an interesting paradox. Recent linear stability analysis at smaller inclination angles[8] shows that there is a critical inclination angle below which a driven contact line solution is linearly stable to transverse perturbations. This critical inclination angle depends on the microscopic length scale at the contact line. On the other hand, experiments[3] show that contact line instabilities occur at smaller inclination angles than those predicted by the linear stability theory. The paper[8] proposed an alternative linear mechanism for the initiation of instabilities. Due to the singular dependence of the base state on the microscopic length scale at the contact line, the linear stability problem also exhibits marked transient-time amplification, with a rate that scales like the microscopic length scale. This study has raised some questions[9–11] regarding whether such transient growth can be observed in experiments.

In this work we clarify the nature of the transient growth, showing that, as was suggested by a heuristic argument in Ref. 8, contact line perturbations, imposed locally, can be amplified by many orders of magnitude. In addition, we find a new phenomenon related to this instability; that there is a characteristic width-scale (larger than the capillary length) for such perturbations to have a significant transient effect. This width-scale is significantly larger than typical surface roughness, implying that special surface design might be needed to reveal this effect experimentally. For ordinary rough surface it would be interesting to determine if compound transient effects could occur.

A problem with similar geometry, involving spin-coating over grooved surfaces, was considered in Ref. 12. They study quasistatic solutions describing planarization over perturbations (trenches) whose depth is of the same order of magnitude as the film thickness. We are interested in a related, but different problem of dynamic stability of driven contact lines flowing over shallow perturbations. Despite these differences, in both situations, the surface inhomogeneities have to be sufficiently wide (in the direction of the flow) in order to influence the shape of the film. We also remark that in a related problem of gravity-Marangoni driven films[13] the microscopic structure of the precursor layer can cause order one changes in the bulk of the film.

Following[8] we consider a dimensionless fourth order nonlinear diffusion equation for the film height, *h:*

$$h_t+\nabla\cdot(h^3\nabla\Delta h-Dh^3\nabla h)+\sin(\alpha)(h^3)_x=0. \qquad (1)$$

Here, the last (convective) term, which arises from the component of gravity in the downstream, *x*, direction is destabilizing, while the diffusion terms tend to stabilize the flow. The dimensionless film height h is h_p/H_N, and H_N is the upstream film thickness, assumed to be a constant (subscript p stands for a value of a quantity in physical units). The downstream film thickness is assumed to be a small constant $\tilde{b}$ associated with the presence of a precursor layer. The dimensionless space and time variables are, respectively, $x=x_p/l_0$, $l_0=(H_N\gamma/\rho g)^{1/3}$, $t=t_pU_0/l_0$, and the velocity scale is $U_0=\rho g H_N^2/(3\mu)$. Also, ρ is the fluid density, μ its dynamic viscosity, γ is the fluid-air surface tension, and α is the inclination angle. The usual capillary length is $l=l_0\sin(\alpha)$ and the average velocity of the contact line is $U=U_0\sin(\alpha)$. The parameter $D=(3\mathrm{Ca}^0)^{1/3}\cos(\alpha)$, and the capillary number is $\mathrm{Ca}=\mu U/\gamma=\mathrm{Ca}^0\sin(\alpha)$. The dimensionless model (1) has upstream boundary condition $h\to1$ as $x\to-\infty$ and $h\to b=\tilde{b}/H_N$ as $x\to\infty$.

It is well known that the competition between stabilizing and destabilizing terms in Eq. (1) can lead to a ''bump'' in the flow profile. The formation of this bump is typically a

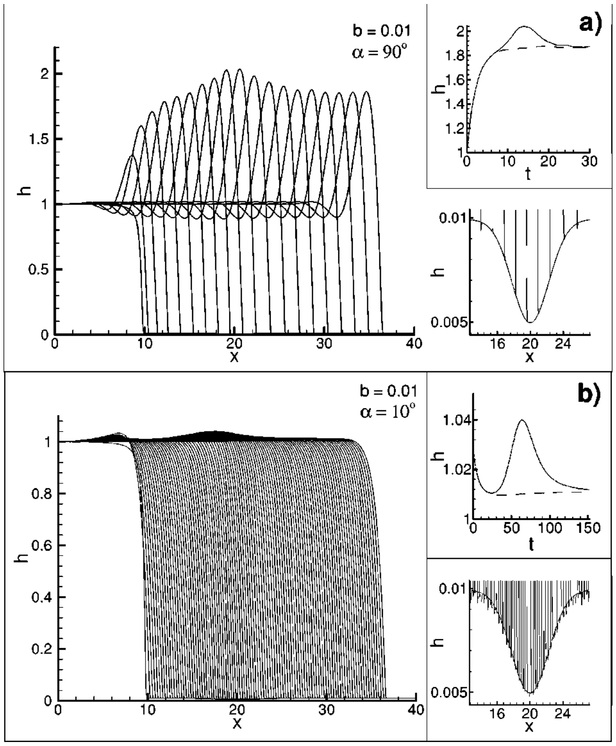

FIG. 1. The flow of a film over perturbation, for $\alpha=90°$ ($D=0$) (a), and $\alpha=10°$ ($D\approx1.61$) (b) [physical parameters correspond to the experiment (Ref. 14)]. The upper inserts show the maximum film height as a function of time, for a perturbed flow (solid line) and unperturbed one (broken line). The lower inserts show the perturbation itself ($s=0.5, w\approx5.3$).

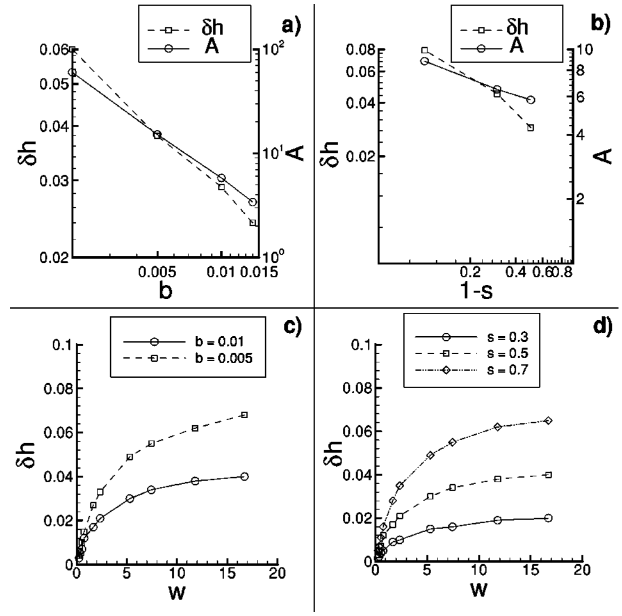

FIG. 3. (a) Amplification A and δh, for different b's ($\alpha=10°$, $s=0.5$, $w=5.3$); (b) δh and A for different s's ($b=0.01,\alpha=10°,w=5.3$); (c) and (d) δh for different w's [(c) $s=0.5$, $\alpha=10°$, (d) $b=0.01,\alpha=10°$].

sign of instability of the flow profile with respect to instabilities in the transverse direction. In order to understand nonlinear stability of Eq. (1), we perform numerical simulations of Eq. (1) in one space dimension and compare these results to the linear stability of Eq. (1) in one and two space dimensions. Linear stability analysis was carried out in Ref. 8, where it was shown that a small perturbation on the scale of b can grow to an order one size. Here we extend the previous work to the fully nonlinear problem and explore in more details the effect of perturbations.

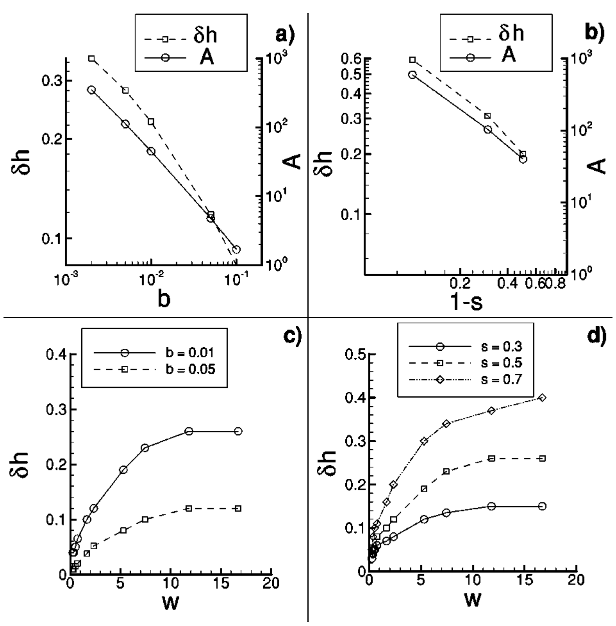

FIG. 2. (a) Amplification A and δh, for different b's ($\alpha=90°$, $s=0.5$, $w=5.3$); (b) δh and A for different s's ($b=0.01,\alpha=90°,w=5.3$); (c) and (d) δh for different w's [(c) $s=0.5$, (d) $b=0.01,\alpha=90°$].

Figures 1(a) and 1(b) show a typical solution of Eq. (1) in the laboratory frame, that develops as a thin film flows down an incline. The perturbation of b of the form $f=-s\exp[-c(x-x_{\mathrm{pert}})^2]$, with characteristic width $w=\sqrt{4\ln 2/c}$ in units of x, and depth s in units of unperturbed b is imposed on the precursor film downstream of the advancing front. We choose such x_{pert} that the fluid is already in steady-state regime (meaning that the unperturbed bump height, h_0, does not change) before it reaches the perturbation, so that the results do not depend on the particular choice of initial conditions. As the film flows over the perturbation, there is a macroscopic change in the bump height, defined as $\delta h=h_{\max}-h_0$, where $h_{\max}$ is the maximum film thickness. As expected, this effect is much stronger for larger α's [compare Figs. 1(a) and 1(b)], since the flow shown in Fig. 1(a) is unstable even without perturbing b, while the steady state behavior of the film in Fig. 1(b) shows stability. Still, a finite size bump is produced even in the case of a stable film, meaning that *instability can be induced by a small perturbation of the precursor thickness.*

Figure 2 shows the results for $\alpha=90°$. Part (a) shows that the effect of a perturbation is stronger for smaller b's. Further, the small perturbation is amplified by the contact line; we observe that the amplification $A=\delta h/(sb)\sim b^{-4/3}$; so the dependence of A on b is even stronger than predicted by the linear theory ($A\sim1/b$).[8] We also note that $\delta h\sim b^{-1/3}$. Figure 2(b) gives the effect of the perturbation depth; δh and A show strong increase for deeper perturbations, which can be understood if one assumes that the ''ef-

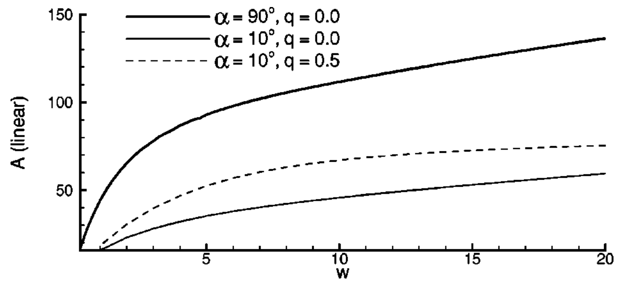

FIG. 4. Transient growth in the solution of the linearized problem. For the strongly unstable situation ($\alpha=90°$), we show only the marginally stable $q=0$ mode, since larger q's are linearly unstable, so that A(linear) is infinite. For $\alpha=10°$, both the $q=0$ and $q=0.5$ modes are shown.

fective'' film thickness is given by $b(1-s)$, in agreement with Refs. 4,8,15.

Figures 2(c) and 2(d) show some surprising results: *the change of film thickness strongly depends on the perturbation width*, *w*. For very narrow *w*, the change in *h* is strongly suppressed; it is more pronounced at the intermediate values; saturation occurs at large *w*'s. The height of the saturation level and its dependence on *b* and *s* can be explained by assuming $\delta h \sim (b(1-s))^{-1/3}$, as mentioned before. Figure 3 shows the results for the flow characterized by small inclination angle, $\alpha=10°$. The general features of the results for $\alpha=90°$ are preserved, even though the amplification of the perturbation is suppressed.

The effect that a perturbation has on the stability of a contact line can be understood as follows. In order to influence the stability of a contact line, the time scale, $\tau_{\mathrm{pert}} \sim w l_0/U$ of the perturbation has to be larger than the time scale on which nonlinear effects enter, $\tau_{\mathrm{nlin}} \sim l/U$. For $w \ll 1$, $\tau_{\mathrm{pert}} \ll \tau_{\mathrm{nlin}}$; so these perturbations do not influence the stability. For $w \gg 1$, the nonlinear effects enter on the time scale $\tau_{\mathrm{nlin}} \ll \tau_{\mathrm{pert}}$, so that the response of the fluid is independent of *w*, leading to the saturation. For the intermediate values of *w*, $\tau_{\mathrm{nlin}} \approx \tau_{\mathrm{pert}}$, resulting in a smooth transition between the two extreme regions. We note that weakly nonlinear analysis of this problem for partial wetting fluid (based on a slip model), also shows strong sensitivity of the contact line instability to variation of the slip length.[16]

Next, we briefly compare the results of fully nonlinear computations with the linear theory, using the same method as in Ref. 8. Expanding $h=h_0+\epsilon e^{iqy}g(x,t)$, we obtain an equation for *g*, which parametrically depends on the wave number *q*. Using the same perturbation as in nonlinear computations, we obtain the transient amplification defined as $A(\mathrm{linear})=\max_{x,t}|g(x,t)|/\max_x|g(x,0)|$. The results are shown in Fig. 4. The effect of the perturbation becomes more pronounced at the same larger *w*'s that are seen for the fully nonlinear problem.

To conclude, we explore the effect which perturbations of a thin film have on the stability of the contact line, and find that small perturbations of the precursor film can have a considerable influence. This effect is augmented for smaller values of precursor thickness, *b*. Such perturbations can be introduced by surface inhomogeneities. We find that the width-scale of the inhomogeneities is of considerable importance; only the perturbations characterized by large enough width-scales modify the flow behavior.

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